H2 Mathematics (9758)

Graphs & Transformations — Exam Notes

A-Level 2027 Syllabus | Chapter 2 | 60% Content / 40% Exam Skills

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Command Words & What They Require

Command Word Sketch

Expectation: A hand-drawn graph showing the correct shape, with all key features labelled. Axes must be drawn with labels (x, y) and a scale indicated. Asymptotes drawn as dashed lines and labelled with their equations. Intercepts labelled with coordinates. Turning points labelled. The graph doesn't need to be perfectly to scale, but relative proportions must be correct (e.g., a maximum must actually look like a maximum).

Command Word State the equations of any asymptotes

Expectation: List each asymptote as an equation on its own line. Format: $x = a$ (vertical), $y = b$ (horizontal), $y = mx + c$ (oblique). One mark per asymptote. No working needed unless deriving from long division.

Command Word Describe the sequence of transformations

Expectation: A clear, ordered list. Use precise language: "translate", "stretch", "reflect". Specify direction and magnitude. E.g.:

- "Translate 3 units in the positive x-direction" or "shift right by 3"
- "Stretch parallel to the y-axis with scale factor 2"
- "Reflect in the x-axis"

Command Word Find the Cartesian equation

Expectation: Eliminate the parameter to obtain an equation in x and y only. Show the elimination steps. State any domain/range restrictions carried over from the parametric form.

Mark Allocation Patterns

Marks	Typical Task
1m	State one asymptote, one intercept, or identify one transformation
2m	Find both asymptotes of a rational function, or find intercepts
3m	Sketch a rational function with asymptotes and intercepts labelled
4m	Full graph sketch with 3+ features, or describe 2–3 transformations in sequence
5–6m	Long question: sketch + find equation of transformed graph + state features
7–8m	Parametric equations: find Cartesian + sketch + state domain/range

Exam Tip Graph sketching marks breakdown

A typical 4-mark sketch question allocates:

- 1m: Correct general shape (branches on correct sides of asymptotes)
- 1m: Asymptotes correctly drawn and labelled
- 1m: Intercepts correctly found and labelled
- 1m: All features shown (turning points, symmetry, correct approach to asymptotes)

Even if your shape is slightly off, you can earn 3 out of 4 from the features alone.

Question Templates

Template 1: Sketch a Rational Function

Typical phrasing: "Sketch the graph of $y = \frac{2x+1}{x-3}$, indicating clearly the equations of any asymptotes and the coordinates of any intercepts."

Method — The 5-Step Sketch:

1. **Vertical asymptote(s):** Set denominator $= 0$, check numerator $\neq 0$. Draw as dashed vertical line, label equation.
2. **Horizontal / oblique asymptote:** Compare degrees. If degree(num) $\leq$ degree(den): $y = \frac{\text{leading coefficient of num}}{\text{leading coefficient of den}}$. If degree(num) $=$ degree(den) $+ 1$: do long division for the oblique asymptote. Draw as dashed line, label equation.
3. **Intercepts:** x-int: set $y = 0$ (numerator $= 0$). y-int: set $x = 0$. Label both with coordinates.
4. **Turning points (if any):** Differentiate using quotient rule, set $f'(x) = 0$, solve. Not always required unless asked.
5. **Test a point** on each side of each VA to determine which branch goes where. Mark these test points lightly or just use them to guide your curve.

Template 2: Describe Transformations in Sequence

Typical phrasing: "Describe a sequence of transformations that maps the graph of $y = f(x)$ to $y = 3f(2x - 4) + 1$."

Method:

1. **Factor inside the bracket:** $3f(2x - 4) + 1 = 3f(2(x - 2)) + 1$
2. **List horizontal transformations (inside-first):**
   - $f(x - 2)$: translate right by 2
   - $f(2x)$: stretch parallel to x-axis with scale factor $\frac{1}{2}$ (compress by 2)
3. **List vertical transformations (outside):**
   - $3f(x)$: stretch parallel to y-axis with scale factor 3
   - $f(x) + 1$: translate up by 1
4. **State the mapping:** $(x, y) \mapsto \left(\frac{x+2}{2},\ 3y + 1\right)$

Warning Alternative sequences

There may be multiple valid sequences, but the standard approach (horizontal first, then vertical) is what examiners expect for method marks. If you do vertical first, the horizontal parameters change — don't mix orders without adjusting.

Template 3: Parametric Equations

Typical phrasing: "A curve is defined by $x = t^2 + 1$, $y = 2t - 3$ for $t \in \mathbb{R}$. Find the Cartesian equation and sketch the curve."

Method:

1. **Express t in terms of x (or y):** From $x = t^2 + 1$, we get $t^2 = x - 1$, so $t = \pm\sqrt{x - 1}$. Or from $y = 2t - 3$, we get $t = \frac{y+3}{2}$.
2. **Substitute into the other equation:** $x = \left(\frac{y+3}{2}\right)^2 + 1 = \frac{(y+3)^2}{4} + 1$
3. **Simplify:** $4(x - 1) = (y + 3)^2$, so $(y + 3)^2 = 4(x - 1)$ — a sideways parabola.
4. **State restrictions:** $x = t^2 + 1 \geq 1$, so domain is $x \geq 1$. Range: $y \in \mathbb{R}$.

Template 4: Modulus and Reciprocal Graphs

Typical phrasing: "Sketch, on separate diagrams, the graphs of $y = |f(x)|$ and $y = \frac{1}{f(x)}$."

Method for $y = |f(x)|$:

1. Sketch $y = f(x)$ lightly (or use the given sketch)
2. Identify portions where $f(x) < 0$
3. Reflect those portions in the x-axis (flip up)
4. Portions where $f(x) \geq 0$ remain unchanged
5. Label any points where the graph meets the x-axis (these are "sharp" corners in the modulus graph)

Method for $y = \frac{1}{f(x)}$:

1. Mark vertical asymptotes where $f(x) = 0$
2. Mark invariant points where $f(x) = 1$ or $f(x) = -1$ (graph passes through these)
3. Where $f(x) \to \pm\infty$: $\frac{1}{f(x)} \to 0$ (approaches x-axis)
4. Where $f(x)$ is positive: $\frac{1}{f(x)}$ is positive. Where $f(x)$ is negative: $\frac{1}{f(x)}$ is negative.
5. Between vertical asymptotes and invariant points: sketch the reciprocal curve. It's above $f(x)$ when $0 < f(x) < 1$, below when $f(x) > 1$.

Answering Techniques

The Graph Sketching Checklist

Before submitting any sketch, verify every item:

Feature	Requirement
Axes	Drawn, labelled x and y, arrowheads
Scale	At least one number on each axis (or intercepts serve as scale)
Asymptotes	Dashed lines, labelled with equation ($x =$, $y =$, or $y = mx + c$)
x-intercepts	Coordinates $(a, 0)$ marked
y-intercept	Coordinate $(0, b)$ marked
Turning points	Coordinates (x, y) marked
Shape	Correct number of branches, correct approach to asymptotes

Using the Mapping Formula

For any combination of transformations $y = af(bx + c) + d$, the point (x, y) on $y = f(x)$ maps to:

$$(x', y') = \left(\frac{x - c}{b},\ ay + d\right)$$

This is the most reliable way to find the image of specific points (intercepts, turning points) under transformations. Find the original point, apply the mapping, and you have the new point.

Example Mapping specific points

Given $f(x)$ passes through $(2, 3)$ and the graph is transformed to $y = 2f(x - 1) + 5$:
$(x', y') = (x + 1,\ 2y + 5)$
So $(2, 3) \mapsto (3, 11)$. The transformed graph passes through $(3, 11)$.

Checking Asymptote Behaviour

To determine which side of an asymptote a branch lies on:

- Pick an x-value just to the right of a VA (e.g., VA at $x = 3$: test $x = 3.1$)
- Evaluate $f(x)$. If $f(3.1)$ is a large positive number, the branch goes to $+\infty$ on that side.
- Check the other side too (test $x = 2.9$).
- This tells you whether each branch approaches $+\infty$ or $-\infty$

Timing Guide

Question Type	Time	Priority
State asymptotes from equation	1–2 min	Quick marks — do first
Sketch rational function (full)	5–7 min	Moderate — methodical process
Describe transformations	3–4 min	Quick — mechanical once you factor
Parametric → Cartesian + sketch	6–8 min	Longer — elimination + domain check
Modulus / reciprocal sketch	4–5 min	Based on given $f(x)$, so read carefully
Combined: transform + sketch + features	8–10 min	Multi-part — flag and return if stuck

Common Errors to Avoid

Warning Error 1: Asymptote mislabelling

Writing "asymptote at 3" instead of "$x = 3$".
Writing "horizontal asymptote: $2x + 1$" when it's actually $y = 2x + 1$ (and it's oblique, not horizontal!).
Always use equation format: $x =$, $y =$, or $y = mx + c$.

Warning Error 2: Forgetting the "+ c" in the shift direction

$y = f(x + 3)$ is a shift **left** by 3, not right.
$y = f(2x)$ compresses the x-coordinates by factor $\frac{1}{2}$ (not 2).
When in doubt, test a point: if $(2, 5)$ is on $f(x)$, where is it on $f(x + 3)$? We need $x + 3 = 2 \Rightarrow x = -1$. The point moved from $x = 2$ to $x = -1$ — left by 3.

Warning Error 3: Incorrect order of transformations

$y = 3f(2x - 4) + 1$: Students often say "shift left 4, then compress by 2".
Correct: factor first! $2x - 4 = 2(x - 2)$. So it's shift right by 2, then compress by factor $\frac{1}{2}$ horizontally, then stretch vertically by 3, then shift up by 1.

Warning Error 4: Missing domain restrictions from parametrics

When converting $x = 2\cos\theta$, $y = \sin\theta$ to Cartesian, you get $\frac{x^2}{4} + y^2 = 1$. But $x = 2\cos\theta$ also means $x \in [-2, 2]$. The ellipse equation without this restriction describes the full ellipse, but the parametric curve might only trace part of it (e.g., if $\theta \in [0, \pi]$, then $y = \sin\theta \geq 0$, so only the upper half). Always check the parameter range.

Warning Error 5: GC "connecting" across vertical asymptotes

Your GC draws a near-vertical line connecting the two branches across a VA. This is a plotting artifact. Your hand-drawn sketch must show a **break** at the VA — the curve approaches the asymptote but does not cross it (unless the factor cancels). Use your mathematical understanding, not the GC display, for asymptotes.

Warning Error 6: Confusing $y = |f(x)|$ with $y = f(|x|)$

$y = |x - 2|$ is a V-shape with vertex at $(2, 0)$ — the part of $y = x - 2$ below the x-axis is reflected up.
$y = |x| - 2$ is $f(|x|)$ applied to $f(x) = x - 2$, giving a V-shape symmetric about the y-axis with vertex at $(0, -2)$.
These are fundamentally different graphs. Draw them to see the difference.

Past-Year Question Patterns

Exam Tip What examiners look for

Based on analysis of past A-Level papers:

- **Rational function sketch:** Appears almost every year in Paper 1. Expect 5–7 marks. Usually asks for asymptotes, intercepts, and sometimes turning points.
- **Transformation sequences:** Frequently paired with a graph sketch. "The graph of $y = f(x)$ is transformed to $y = g(x)$. Describe the transformations and sketch $g(x)$" — 6–8 marks.
- **Parametric equations:** Appears in roughly 1 in 2 papers. Usually 6–8 marks: find Cartesian, sketch, state domain/range.
- **Modulus and reciprocal:** Tested as a sub-part of a larger transformation question. 3–4 marks for sketching $|f(x)|$ or $1/f(x)$.
- **Conic sections:** Less frequent but high-stakes when they appear. Ellipse or hyperbola sketch may carry 5–6 marks.